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Eccentricity and centrality in networks

Per Hage [a,*], Frank Harary [b]

[a] Department of Anthropology, University of Utah, Salt Lake City, UT 84112, USA
[b] Department of Computer Science, New Mexico State University, Las Cruces, NM 88003, USA

Abstract

The classic concept of centrality discovered by Camille Jordan in the 19th century is introduced as a model for social network analysis. It is generalized to include the path center of a graph and illustrated with an application to two island networks in Oceania. It is shown to be a necessary addition to the concepts of degree, closeness and betweenness centrality as distinguished by Freeman.

1. Introduction

In a conceptual clarification of centrality models in social network analysis, Freeman (1979) distinguishes between degree centrality, closeness centrality and betweenness. All three models have proven useful in anthropology, with applications to the study of power in informal exchange networks (Hage and Harary, 1981, 1983) and economic success and social stratification in trade networks (Irwin, 1983; Hunt, 1988; Kirch, 1988; Peregrine, 1991; Hage and Harary, 1991; Broodbank, 1993; Milicic, 1993). Recent research has shown the need for refinement of these models in applications to weighted graphs (Freeman et al., 1991) and oriented graphs (White, no date). There is also a need for new centrality models. One of these uses the classic unmarked definition of centrality in graphs first discovered by the great French mathematician Camille Jordan (1869). In the literature of graph theory (Harary, 1969) this is simply called 'centrality', as opposed to median centrality (Buckley and Harary, 1990). The model of the 'center of a graph' is well known in operations research (OR) and has clear applications to social network studies.

* Corresponding author.

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In OR, a typical problem is that of choosing a site for a facility on the basis of a specific criterion (Slater, 1981; Buckley, 1987). In some cases the problem is to choose a site which minimizes the travel time between a site and all other locations. In other cases the problem is to minimize the response time to any other location. When modeled as a graph, the first problem is solved by finding the set of nodes whose total distance to all other nodes is least, i.e. the median of the graph. The second problem is solved by finding the set of nodes whose maximum distance to any other node is least, i.e. the 'center' of the graph. An analogue of the second problem occurs in certain island networks in Oceania.

2. Definitions

A *graph* G consists of a finite non-empty set $V = V(G)$ of *nodes* together with a set $E = E(G)$ of *edges* joining certain pairs of distinct nodes of G. A *path* in G is an alternating sequence $v_0, e_1, v_1, e_2, v_2, \ldots, v_{n-1}, e_n, v_n$ of distinct nodes and edges. A *cycle* is obtained from a path when the initial and terminal nodes v_0 and v_n are joined by an edge. Graph G is *connected* if every pair of nodes are joined by a path. The *length* of a path is the number of edges in it. The *distance* $d(u, v)$ between nodes u and v is the length of a shortest path joining u and v. We write d_{ij} for $d(v_i, v_j)$.

In a connected graph G the *distance sum* or *status of a node* u, written $s(u)$, is the sum of the distances between u and all other nodes (Harary, 1959). Thus

$$s(v_i) = \sum_{j=1}^{p} d_{ij}$$

The *median* of G, also called its *distance center*, is the set of all nodes u of G such that $s(u)$ is minimum (Buckley and Harary, 1990). In Fig. 1, the median is node c with $s(u) = 8$.

The *eccentricity*, $e(v)$, of a node v in a connected graph G is the maximum distance $d(v, u)$ for all u. The *diameter* of a graph G is the maximum eccentricity of a node, i.e. the maximum distance between two nodes of G. The *radius* $r(G)$ is

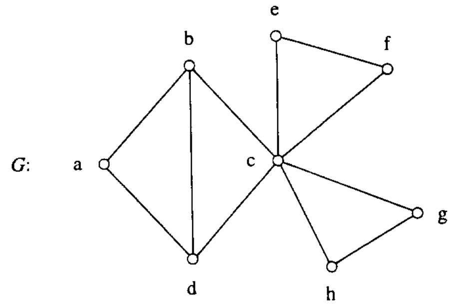

Fig. 1. A graph G.

the minimum eccentricity of the nodes. A node is *central* if $e(v) = r(G)$ and the *center* of G is the set of all central nodes. In Fig. 1, the center of G consists of nodes b, c, d, with $e(v) = 2$.

3. Application

The Marshall Islands in eastern Micronesia are divided into two atoll chains, Ralik and Ratak. At various times in Marshallese history the islands in each chain were conquered and ruled over by paramount chiefs (Mason, 1947). Chiefs controlled their island networks by exploiting ties of kinship and marriage, by monopolizing the means of communication (canoe travel and technology), and by choosing as their capitals strategically located islands from which political pressure and military force could be most readily applied. As measured in inter-island hops, their concern was not the total distance from the home island to all other islands, but the maximum distance to any other potentially rebellious island. Hence they chose the center rather than the median of the graph of the inter-island voyaging network.

Fig. 2 shows graphs of the voyaging networks of the Ratak and Ralik chains constructed from data in Erdland (1910, 1914) and Winkler (1901). The center of the Ratak graph consists of three nodes, Aur, Maloelap and Wotje with $e(v) = 3$, and the center of the Ralik graph consists of a single node, Namu, which also has $e(v) = 3$. The capital of Ratak was Aur, and the capital of Ralik was Namu. These two islands were also symbolically important choices: in Marshallese mythology they were identified as the 'mother of all the clans' in each chain. Notice that the median of each graph is not identical with the center: the median of the Ratak graph is Wotje with $s(u) = 14$, and that of the Ralik graph is Kwajalein with $s(u) = 20$. Only the concept of the center predicts the politically and symbolically most important islands in each network.

4. Discussion

In the Ratak graph both central nodes are in the same 'block'. Such will always be the case. The following three theorems provide useful information concerning the structural properties of centrality in a graph. We require a few more definitions.

A *tree* is a connected graph with no cycles (acyclic). A *cutnode* of a connected graph is a node whose removal together with its incident edges results in a disconnected graph. A *non-separable* graph is connected, nontrivial and has no cutnodes. A *block* of a graph is a maximal nonseparable subgraph. In Fig. 1, c is a cutnode and there are three blocks: $(abcd)$, (cef), (cgh). The classical theorem of Jordan (1869) determines the location of the center when the given graph is a tree.

Theorem 1. The center of a tree consists of either a single node or a pair of adjacent nodes.

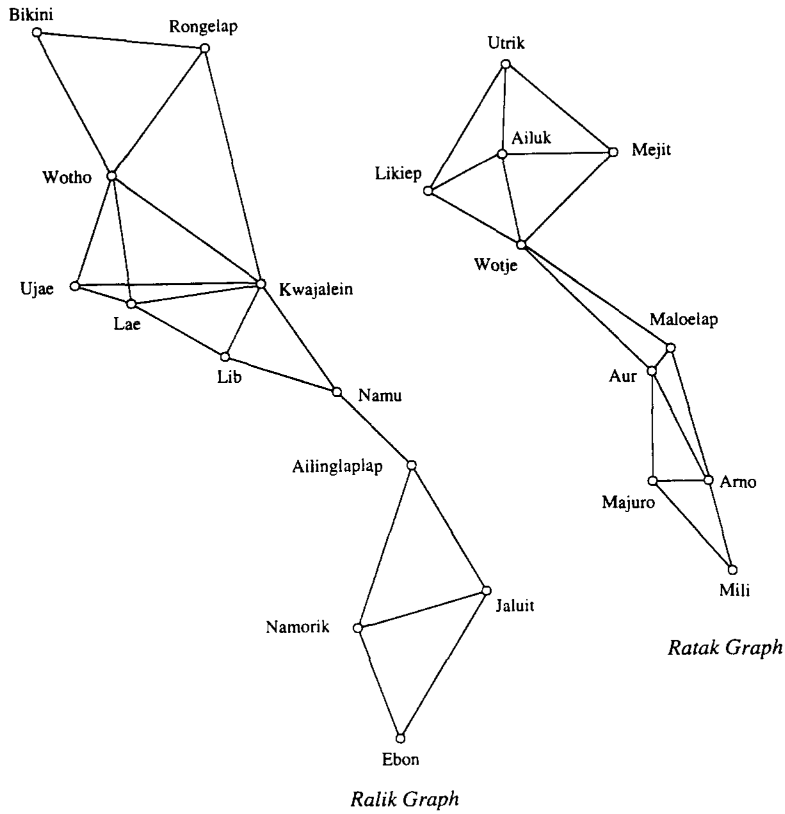

Fig. 2. Graphs of the Marshall Islands voyaging network.

The proof is in Harary (1969). This is illustrated in Fig. 3 with the central nodes labelled (x). To find the center of a tree T, we first remove the endnodes of T. (This process is called 'pruning the tree' in computer science.) In the resulting subtree T', called the derivative of T (Harary 1988), the eccentricity of each node has been decreased by 1. We then continue the process, obtaining successively smaller trees having the same center as T until either a single node or two adjacent nodes remain.

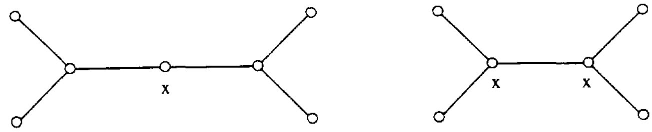

Fig. 3. Trees with one and two central nodes.

Theorem 2. The center $C(G)$ of any connected graph G lies within a block of G.

The proof was given by Harary and Norman (1953) in this generalization of Theorem 1 from trees to graphs. This is illustrated in Figs. 1 and 2, as already indicated.

Buckley has eloquently described the utilization of this result.

> Theorem 2 is sometimes used as the basis for algorithms to find the center of graphs other than trees when the graph has several cutnodes, all of which are known. The theorem enables one to begin by finding the eccentricity of each cutnode. The center must be contained in a block incident with a cutnode of minimum eccentricity. Thus, the eccentricity is found only for nodes in blocks incident with cutnodes with minimum eccentricity (Buckley, 1987: 28).

In some anthropological applications it may be possible as well as desirable to model a network as a *network*, i.e. a weighted graph. The center of a network N is defined as that of its underlying graph G, obtained by ignoring the weights on the edges of N.

Theorem 3. The center of any network lies in a single block.

In some problems in facilities location research it is required to find a path that all nodes are 'close to' (Buckley and Harary, 1990). An anthropological analogue would be finding a sequence of islands that all other islands are close to – the 'backbone' of the network, formally, the 'path center' of a graph now defined formally.

Let W be a subgraph of a given graph G. For any node v, the *distance* $d(v, W)$ from v to W is the minimum distance from v to a node in W. The *eccentricity* of W, $e(W)$, is the distance to a node farthest from W. Thus $e(W) = \max d(v, W)$ for v in G. We restrict our attention to the situation where W is a path in G. A path P is a *path center* of G if P has minimum eccentricity and has minimum length among such paths. For the tree in Fig. 4, paths *gfdik* and *abcdf* have eccentricity 3 and 2, respectively. The central path is *cd* with eccentricity 2.

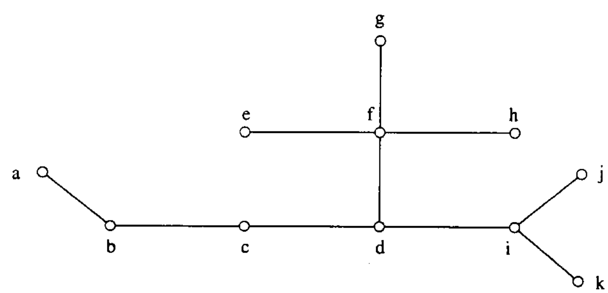

Fig. 4. A tree to illustrate central paths.

In the Ralik graph in Fig. 2, paramount chiefs maintained a residence at Ailinglaplap as well as Namu. In fact Ailinglaplap was sometimes erroneously identified as the capital of Ralik (Pollock, 1970). If chiefs had been similarly established at Kwajalein they would have been at most two steps away from any potentially rebellious breakaway island. These three islands constitute a path center of the Ralik graph.

5. Conclusion

As network analysis gains momentum in anthropology and archaeology, we expect that the repertoire of centrality models will continue to expand. For example, in a forthcoming work (Hage and Harary, 1995) we found it necessary to define 'betweenness in a rooted graph' to handle networks in which the economic success of all communities depends on their intermediate position with respect to a dominant or source community. Such is the case in the trade networks of the Torres Strait in Melanesia. An extensive survey of applicable centrality concepts is given in Buckley and Harary (1990, Chapter 2).

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